Pseudospectral approximation of eigenvalues of derivative operators with non-local boundary conditions

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Abstract

By taking as “prototype problem” the following linear differential system with discrete delay
\[
\begin{cases}
x'(t) = L_0 x(t) + L_1 x(t - \tau), & t \geq 0 \\
x(t) = \phi(t), & -\tau \leq t \leq 0
\end{cases}
\]
where \(L_0, L_1 \in \mathbb{C}^{m \times m}\) and \(\tau > 0\), we will present the reformulation of a retarded functional differential equation as the abstract Cauchy problem on \(X = C([-\tau, 0], \mathbb{C}^m)\)
\[
\begin{cases}
y'(t) = A y(t), & t \geq 0 \\
y(0) = \phi
\end{cases}
\]
where \(y : [0, +\infty) \rightarrow D(A) \subseteq X, \phi \in D(A)\) and \(A : D(A) \rightarrow X\) is a derivative operator which satisfies suitable boundary conditions given by the particular system (1) considered and contained in the domain \(D(A)\).

The asymptotic behavior of the solutions of (1) is completely described by the eigenvalues of the operator \(A\). In particular the zero solution of (1) is asymptotically stable if and only if all the eigenvalues of \(A\) have strictly negative real part.

The analysis of the spectrum of derivative operators with general non-local boundary conditions represents an important tool for the investigation of the stability properties of solutions for more general classes of equations, which are important for applications: linear autonomous differential systems with multiple discrete and distributed delays, equations modeling age-structured population dynamics, reaction diffusion equations with delay in the reaction term, equations of mixed type (i.e. advanced-retarded equations), neutral delay equations.
It is thus relevant to have a numerical technique to approximate the eigenvalues of derivative operators $\mathcal{A}$ under non-local boundary conditions. We propose to discretize the operators $\mathcal{A}$ by pseudospectral methods and turn the original problem into a suitable matrix eigenvalue problem. This approach is particularly efficient due to the well-known "spectral accuracy" convergence of pseudospectral methods.