State Dependent Delay and Advanced-Delay Differential Equations

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Wheeler-Feynman Action-at-a-distance Electrodynamics

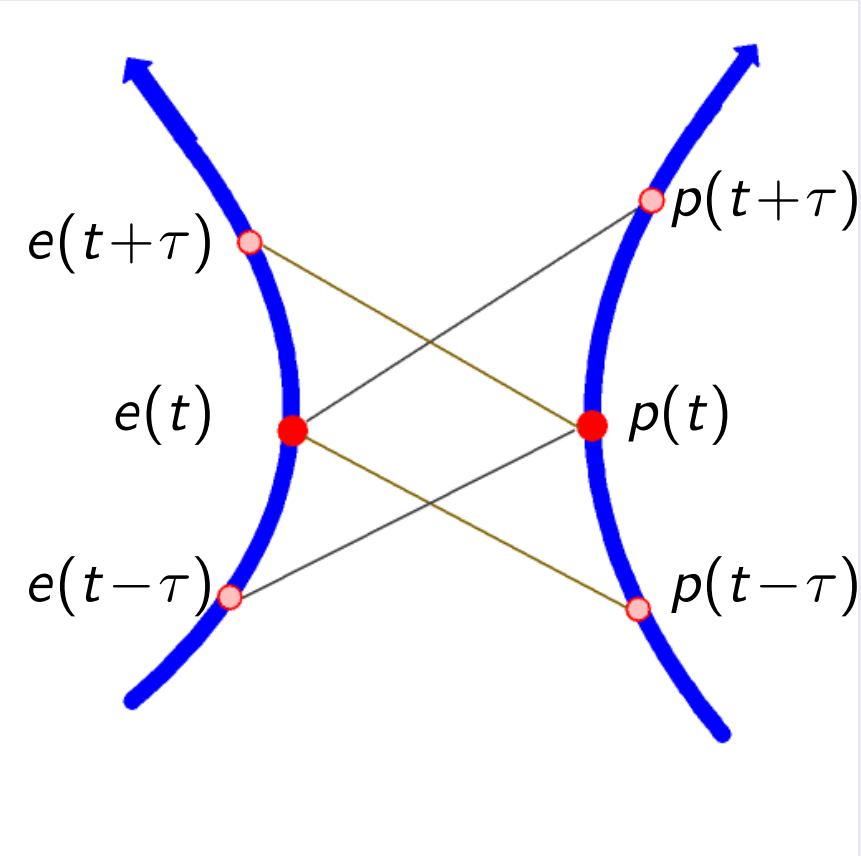
In his 1965 Nobel Prize Lecture Richard Feynman explained how this problem launched his research but lamented to some extent abandoning the original problem along the way.

Note

John Wheeler's name comes first as he was Feynman's supervisor. Wheeler, who just died in 2008, was a very distinguished physicist, who also coined the term *black hole*. (And he was a much more successful supervisor than Feynman)

Action-at-a-distance Electrodynamics
Implicitly State Dependent Advance-Delay DEs

Implicit State-Dependent Delay



$$m_e\ddot{e} = F_{ep} = \tfrac{1}{2}(F^+_{ep} + F^-_{ep})$$

$$m_p\ddot{p} = F_{pe} = \tfrac{1}{2}(F^+_{pe} + F^-_{pe})$$

$$F^{\pm}_{ep}(t) = -K\frac{p(t\pm\tau) - e(t)}{|p(t\pm\tau) - e(t)|^3}$$

$$F^{\pm}_{pe}(t) = -K\frac{e(t\pm\tau) - p(t)}{|e(t\pm\tau) - p(t)|^3}$$

$$|p(t\pm\tau) - e(t)| = c\tau$$

$$|e(t\pm\tau) - p(t)| = c\tau$$

[Wheeler, Feynman 1945 & 1949], [Schild 1963],[Many Others...]

Example
Travelling Waves for Discrete Nagumo Equation

Discrete Nagumo Equation

$$\dot{u}_i = (u_{i+1} - 2u_i + u_{i-1}) - \beta f(u_i),\ f(u) = u(u-a)(u-1)$$

Example
Travelling Waves for Discrete Nagumo Equation

Travelling Wave ansatz $u_i(t) = \varphi(i - ct) = \varphi(\xi)$ gives

$$-c\varphi'(\xi) = \varphi(\xi + 1) - 2\varphi(\xi) + \varphi(\xi - 1) - \beta f(\varphi(\xi))$$

Conclusions

Delay Differential Equations (without advances)

Also of interest and easier! Arise in all applications but especially engineering and biology.

Explicit State Dependent Problems

Consider

$$\dot{u}(t) = -u(t) - \kappa u(t - a - cu(t)), \qquad \kappa, a, c > 0$$

Here delay $= a + cu(t)$ is explicit (as opposed to WF where delay depends on solution).
Even these problems not yet understood.

There is plenty of work still to be done.....