Applied Mathematics

McGill & CRM Applied Mathematics Seminar

2:35 pm Monday 8 December 2003
At McGill, Burnside Hall 1205

"Rossby Solitary Wave in the Presence of a Critical Layer"

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Coffee and refreshments will be served after the seminar

Abstract: We consider the evolution of long Rossby waves in a horizontally sheared zonal current. We restrict attention to a stable flow so that the nonlinear time scale is long. This enables flow to organize itself into a large-scale coherent structure in the regime where a competition sets in between weak nonlinearity and weak dispersion. This balance is often described by a Korteweg-de-Vries equation (KdV). The traditional assumption of a weak amplitude breaks down when the wave speed equals the mean flow velocity at a certain latitude, due to the appearance of a singularity in the leading order equation, which strongly modifies the flow in a critical layer. Nonlinear effects are invoked to resolve this singularity, since the relevant geophysical flows are of high Reynolds number. Viscosity is also introduced in order to render the nonlinear critical layer solution unique, but the inviscid limit is eventually taken. By the method of matched asymptotic expansions, this inner flow will be matched at the edges of the critical layer with the outer flow. We show that the critical-layer induced flow leads to a strong rearrangement of the related streamlines and consequently of the potential vorticity contours, particularly in the neighbourhood of the separatrices bounding the open and closed streamlines. The symmetry of the critical layer vis-a-vis the critical level is also broken. This theory is thus relevant for the phenomenon of Rossby wave breaking and eventual saturation into a nonlinear wave. Spatially localized solutions are described by a KdV equation which is strongly modified by functionals of the amplitude and its temporal derivative which depend on the critical-layer shape, that is depression or elevation waves. These additional terms are made necessary at a certain order of the asymptotic expansion while matching the inner flow on the dividing streamlines. The new evolution equation is not in general integrable, but supports a family of solitary waves.