hp-ADAPTIVE FINITE ELEMENTS
a Quest for Exponential Convergence

L. Demkowicz

Institute for Computational Engineering and Sciences
The University of Texas at Austin

I will review the fundamentals and discuss the current state of development of fully automatic hp-adaptive conforming finite elements. The H^1-, $H(\mathrm{curl})$- and $H(\mathrm{div})$-conforming hp elements extend the classical elements of Nédélec, Raviart-Thomas, and Brezzi-Douglas-Fortin-Marini, forming a complete family of discrete spaces constituting the de Rham diagram,

$$\begin{array}{ccccccc} H^1 & \xrightarrow{\nabla} & \boldsymbol{H}(\mathrm{curl}) & \xrightarrow{\nabla\times} & \boldsymbol{H}(\mathrm{div}) & \xrightarrow{\nabla\circ} & L^2 \\ \big\downarrow\Pi & & \big\downarrow\Pi^{\mathrm{curl}} & & \big\downarrow\Pi^{\mathrm{div}} & & \big\downarrow P \\ W_{hp} & \xrightarrow{\nabla} & \boldsymbol{Q}_{hp} & \xrightarrow{\nabla\times} & \boldsymbol{V}_{hp} & \xrightarrow{\nabla\circ} & Y_{hp} \end{array} .$$

The hp discretizations have been constructed for tetrahedra, prisms and hexahedra. The *projection-based* interpolation operators $\Pi, \Pi^{\mathrm{curl}}, Pi^{\mathrm{div}}$ are defined in an identical way for all families of elements, and provide a foundation for stability and convergence analysis, see [4, 1, 3] for details. The projection-based interpolation has also turned out to be the key for developing a succesful, fully automatic hp-adaptive algorithm that produces a sequence of hp meshes delivering exponential convergence for both elliptic and Maxwell problems [2].

I will present and discuss the key components of the hp methodology, provide an overview of currently available results, and list open problems.

References

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Leszek F. Demkowicz is Assistant Director of the Institute for Computational Engineering and Sciences (ICES) and a Professor in the Department of Aerospace Engineering and Engineering Mechanics, at the University of Texas at Austin. He has a M.S. in mathematics from Jagiellonian University, and M.S. (Hons.), Ph.D. and Sc.D. degrees in mechanics from Cracow University University of Technology, Poland. Prior to joining the University of Texas faculty, he held various positions at Cracow University University of Technology. In years 1990-1993, he was the head of Section of Applied Mathematics and Director of Computer Center.

Dr. Demkowicz authored a monograph on adaptive methods (in Polish), co-authored with Prof. J.T. Oden a textbook on Functional Analysis (CRS Press, 1996), and co-edited two books. Dr. Demkowicz has also authored over 120 journal articles, conference proceedings, book chapters and technical reports in the general area of computational mechanics and mathematics. He is Associate Editor of four international journals. Dr. Demkowicz was the founding member of Polish Association for Computational Mechanics and served as its first President. He is a fellow of both U.S. and International Associations for Computational Mechanics and a member of several other professional organizations. He graduated 8 Ph.D. and numerous M.S. students.

His work and scientific interests span across numerical analysis, adaptive finite element methods, and wave propagation problems, including acoustics, elastodynamics and electromagnetics. Among other applications, Dr. Demkowicz and his group developed original numerical methods for structural vibrations, analysis of acoustics of human ear, dynamic modeling of gears, analysis of optical waveguides, calculation of Radar Crossections, borehole electromagnetics and acoustics. He has given numerous invited talks on the subjects. His work has been sponsored by NSF, Navy, Air Force, DOE, Schlumberger, Baker-Hughes and Boeing. His research on high accuracy adaptive methods has recently been summarized in a book - *Computing with hp Elements* (Francis & Taylor, in press). He has also recently got involved with new applications including a multiscale modeling of imprint lithography (with J.T. Oden and G. Wilson) and bioengineering applications.