Applied Mathematics

McGill & CRM Applied Mathematics Seminar

2:35 pm Monday 19th January 2004
At McGill, Burnside Hall 1205

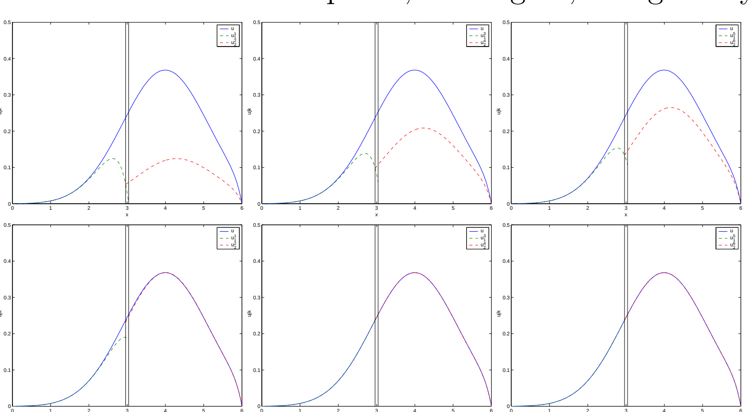
“Modern Waveform Relaxation Methods”

Martin J. Gander
Department of Mathematical Sciences
McGill University

Coffee and refreshments will be served after the seminar

Abstract: The Piccard-Lindeloef iteration from 1890 is an elegant analytical tool to prove existence and uniqueness of solutions to ordinary differential equations. Ninety years later a team led by Ruehli at IBM Watson, Yorktown heights, invented an algorithm to solve their extremely large systems of ordinary differential equations coming from the VLSI design of their next generation of processors: waveform relaxation. Only a decade later, Nevanlinna analyzed the link between waveform relaxation and the Piccard-Lindeloef iteration, and proved super-linear convergence of the waveform relaxation method under very mild conditions on the differential equations.

Starting with the historical proof of convergence of Piccard, I will show how Ruehli and his team invented waveform relaxation, Nevanlinna proved convergence, and then show why these methods in their historical form are not suitable for large scale computations: the convergence is too slow. I will then generalize the waveform relaxation method to solve partial differential equations and show how the algorithm, through a small modification, becomes a very fast, parallel solver. I will prove for one algorithm in the new class that it is well posed, convergent, and give asymptotic convergence results of practical interest.



Top series of pictures shows how iterates of the Piccard-Lindeloef or classical waveform relaxation iteration converge, whilst in the the lower series by the second iteration already, iterates of the modern waveform relaxation method cannot be distinguished from the exact solution.