MATH316 Midterm Test 1 – (Oct 22, 2007)

Problem 1. [6], Find the square roots of $z = -2 + 2\sqrt{3}i$ and express them in rectangular coordinates.

Problem 2.

- (a.) [7] Show that $u(x, y) = Log(x^2 + y^2)$ is harmonic in $\mathbb{C} \setminus \{0\}$. (b.) [9] Let $\Omega = \mathbb{C} \setminus \{z = x + iy | x = 0, 0 < y < \infty\}$, can you find a harmonic conjugate function v of u in Ω ? If so, write down v; if not, provide a reason.
- $(c^*.)$ Can you find a harmonic conjugate function v of u in $\mathbb{C} \setminus \{0\}$? If so, write down v; if not, provide a reason. (* This is a bonus question.)

Problem 3. Let C denote the positively oriented boundary of the square $|x| \leq 2, |y| \leq 2$, use Cauchy's formulas to evaluate the following integrals:

(a.) [7] $\int_C \frac{\cos(e^z) + z^8}{z} dz;$ (b.) [7] $\int_C \frac{\sin(5z)dz}{(z-i)^2}.$

Problem 4. [14], Find power series expansion of function $f(z) = \frac{z^3}{(i-z)^2}$ around the points $z_0 = 0$ and $z_0 = 1$, determine the corresponding radius of convergence in each case.