Fourth Assignment, due on October 30, 2017.

Problem 1 [10], Let f be continuous on closed disc $D_1 = \{x^2 + y^2 \leq 1\}$. Denote $D_r = \{x^2 + y^2 \leq r^2\}$. Prove that

$$\lim_{r \to 0} \frac{1}{area(D_r)} \int\int_{D_r} f(x,y) dA = f(0,0).$$

Problem 2 [10], Given that the double integral $\int\int_D f(x,y)dxdy$ of a positive continuous function f equals the iterated integral $\int_0^1 [\int_{x^2}^x f(x,y)dy]dx$, sketch the region D and interchange the order of integration.

Problem 3 [10], Find the volume of the region common to the intersecting cylinders $x^2 + y^2 \leq a^2$ and $x^2 + z^2 \leq a^2$.

Problem 4 [10],

(a.) Sketch the region for the integral $\int_0^1 \int_0^x \int_0^y f(x,y,z)dzdydx$.

(b.) Write the integral with the integration order $dxdydz$.

Problem 5 [10], For the region $W = \{(x,y,z) || x| \leq 1, |y| \leq 1, z \geq 0$ and $x^2 + y^2 + z^2 \leq 1\}$, find the appropriate limits $\phi_1(x), \phi_2(x), \gamma_1(x,y)$ and $\gamma_2(x,y)$, and write the triple integral over W as iterated integral in the form

$$\int\int\int_W f dV = \int_a^b \{\int_{\phi_1(x)}^{\phi_2(x)} [\int_{\gamma_1(x,y)}^{\gamma_2(x,y)} f(x,y,z)dz]dy\}dx.$$

Problem 6 [10], Calculate $\int\int_R \frac{1}{x+y} dydx$, where R is the region bounded by $x = 0, y = 0, x + y = 1, x + y = 4$, by using the mapping $T(u,v) = (u - uv, uv)$.

Problem 7 [10], Let D be the unit disk. Express $\int\int_D (1 + x^2 + y^2)^{\frac{5}{2}} dxdy$ as an integral over $[0,1] \times [0, 2\pi]$ and evaluate.

Problem 8 [10], Use spherical coordinates to evaluate

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \frac{\sqrt{x^2+y^2+z^2}}{1 + [x^2+y^2+z^2]^2} dzdydx.$$

Problem 9 [10], Evaluate $\int\int_D e^{(x+y)^2} dxdy$, where D is the interior of the triangle with vertices $(0,0), (1,3)$, and $(2,2)$.

Problem 10 [10], Evaluate improper integral $\int\int\int_{\mathbb{R}^3} f(x,y,z)dxdydz$, where

$$f(x,y,z) = \frac{1}{[1 + (x^2+y^2+z^2)^{\frac{3}{2}}]^{\frac{9}{2}}}.$$