third Assignment, due on October 17, 2014.

Problem 1 [10], Suppose u is a C^2 function in $\mathbb{R}^2$ satisfying equation $\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} \geq 0$, for all $(x, y) \in \mathbb{R}^2$. Prove that, if u has a local maximum $(x_0, y_0) \in \mathbb{R}^2$, then u is a constant function.

Problem 2 [10], Let $\mathcal{S} = \{x \in \mathbb{R}^n | g(x) = 0\}$ be a level surface of a differentiable function g in $\mathbb{R}^n$. Suppose $x_0 \in \mathcal{S}$ such that $\|x_0\| \geq \|x\|$ for all $x \in \mathcal{S}$, show that $x_0 = \lambda \nabla g(x_0)$ for some $\lambda \in \mathbb{R}$.

Problem 3 [10], Show that $xy + z + 3xz^5 = 4$ is solvable for z as a function of (x, y) near $(1, 0, 1)$. Compute $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ at $(1, 0)$.

Problem 4 [10], Show that there are positive numbers p and q and unique function u and v from interval $(-1 - p, -1 + p)$ into the interval $(1 - q, 1 + q)$ satisfying

$$xe^{u(x)} + u(x)e^{v(x)} = 0 = xe^{v(x)} + v(x)e^{u(x)}$$

for all x in the interval $(-1 - p, -1 + p)$ with $u(-1) = v(-1) = 1$.

Problem 5 [10], Suppose $f, g, h : \mathbb{R} \to \mathbb{R}$ are differentiable. Show that the vector field $\mathbf{F}(x, y, z) = (f(x), g(y), h(z))$ is irrotational.

Problem 6 [10], Find the arc length of $\mathbf{c}(t) = t\mathbf{i} + (\log t)\mathbf{j} + 2\sqrt{2t}\mathbf{k}$ for $1 \leq t \leq 2$.

Problem 7 [10], Evaluate the integral $\int\int_R (ax + by + c)dxdy$ for $R = [0, 1] \times [0, 1]$.

Problem 8 [10], Let f be continuous on $R = [a, b] \times [c, d]$; for $a < x < b, c < y < b$, define

$$F(x, y) = \int_a^x \int_c^y f(u, v)dudv.$$

Show that $\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial^2 F}{\partial y \partial x} = f(x, y)$. Use this example to discuss the relationship between Fubini's theorem and the equality of mixed partial derivatives.

Problem 9 [10], Evaluate $\int_0^1 \int_0^{x^2} (x^2 + xy - y^2)dydx$. Describe this interated integral as an integral over a certain region D in the xy plane.

Problem 10 [10], Find $\int_0^4 \int_{y/2}^2 e^{x^2} dxdy$.