Due on October 3.

Problem 1 [10], Compute an equation for the plane tangent to the graph of $f(x,y) = \frac{\sin(x^2+y^2)}{\cos(x+y)}$ at $x = -1, y = 1$.

Problem 2 [10], Let f and g be functions from $\mathbb{R}^3$ to $\mathbb{R}$. Suppose f is differentiable and $\nabla f(\mathbf{X}) = g(\mathbf{X})\mathbf{X}, \forall \mathbf{X} \in \mathbb{R}^3$. Show that $\forall \mathbf{X}, \mathbf{Y} \in \mathbb{R}^3$, if $\|\mathbf{X}\| = \|\mathbf{Y}\|$, then $f(\mathbf{X}) = f(\mathbf{Y})$. That is, the restriction of f on any sphere centred at the origin is a constant function.

Problem 3 [20], Let $f(x,y) = \frac{xy(y^2-x^2)}{x^2+y^2}$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$.
(a), If $(x,y) \neq (0,0)$, calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
(b), show that $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0$.
(c), show that $\frac{\partial^2 f}{\partial x \partial y}(0,0) \neq \frac{\partial^2 f}{\partial y \partial x}(0,0)$.
(d), what went wrong? Why are the mixed partial derivatives are not equal?

Problem 4 [10], A function $f : \mathbb{R} \to \mathbb{R}$ is called an analytic function provided that, $\forall x \in \mathbb{R}$, there is $\delta > 0$ such that $\forall |h| < \delta$,

$$f(x+h) = f(x) + f'(x)h + \cdots + \frac{f^{(k)}(x)}{k!}h^k + \cdots = \sum_{k=0}^{\infty} \frac{f^{(k)}(x)}{k!}h^k$$

the series on the right-hand side converges and equals $f(x+h)$. Prove that, if f satisfies the following condition: On any closed interval $[a,b]$, there is a constant M (depending on $[a,b]$) such that for all $k = 1, 2, \cdots$, $|f^{(k)}(x)| \leq kM^k$ for all $x \in [a,b]$, then f is analytic.

Problem 5 [10], Let $f(x,y,z) = x^2 + y^2 + z^2 + \alpha xy$.
(a) Verify the $(0,0,0)$ is a critical point for f.
(b) Find all values of α such that f has a local minimum at $(0,0,0)$.

Problem 6 [10], Show that if $x_0 = (x_1^0, x_2^0)$ is a critical point of a C^3 function f and $f_{11}(x_0)f_{22}(x_0) - f_{12}^2(x_0) < 0$, then there are points x and $\tilde{x}$ near x_0 such that $f(x) > f(x_0)$ and $f(\tilde{x}) < f(x_0)$.

Problem 7 [10], Find the absolute maximum and minimum value for the function $f(x,y) = \sin(xy)$ on the rectangle R defined by $-1 \leq x \leq 1, -1 \leq y \leq 1$.

Problem 8 [10], Find the absolute maximum and minimum values for the function $f(x,y,z) = x^2 + y^2 + z^2 + z + xy$ on the ball $B = \{(x,y,z) | x^2 + y^2 + z^2 \leq 1\}$.

Problem 9 [10], Find the maximum and minimum of $f(x,y) = xy - x + y - 1$ on the set $x^2 + y^2 \leq 4$.