Sample of problems

(1) Interchanging the orders of integration for the double integral $\int_0^{\frac{\pi}{2}} \int_0^{\cos x} y \sin x \, dydx$ gives:
  (a) $\int_0^{\sin x} y \; dy \int_0^{\cos y} \sin x \; dx$,
  (b) $\int_0^{\cos y} \sin x \; dx \int_0^1 y \; dy$,
  (c) $\int_{y=0}^{\frac{\pi}{2}} \int_0^{\cos^{-1} x} y \sin x \; dxdy$,
  (d) $\int_0^1 \int_0^{\cos^{-1} y} y \sin x \; dx \; dy$,
  (e) $\int_0^1 \int_0^{\cos y} y \sin x \; dxdy$.
  Answer: (d)

(2) Let $x(y, z)$ be defined implicitly by $xy^3 = 2y - z$. Then $\left(\frac{\partial x}{\partial y}\right)^2 + \left(\frac{\partial x}{\partial z}\right)^2$ at the point $(0, 1, 2)$ has the value
  (a) -0.2, (b) 0.2, (c) 5, (d) 2, (e) -0.5.
  Answer: (c)

(3) Define $z = Z(x, y)$ implicitly by $x^2 + xy + yz + z^2 = 3$ with $Z(1, 2) = 0$. The directional derivative at $(1, 2)$ in the direction of the vector $(3, -4)$ is
  (a) -0.8, (b) -8, (c) -2.8, (d) 2.8, (e) 0.8.
  Answer: (a)

(4) Of the two infinite series

$$A : \sum_{n=1}^{\infty} (-1)^n \frac{e^{-n^2}}{\ln(n+1)}, \qquad B : \sum_{n=1}^{\infty} \frac{\sqrt{n^2+4}}{n^2}$$

  (a) both are absolutely convergent
  (b) both are divergent
  (c) A is divergent and B is convergent
  (d) A is absolutely convergent and B is divergent
  (e) A is absolutely convergent and B is conditionally convergent
  Answer: (d)

(5) The surface $z = Z(x, y)$ is defined implicitly by $x^2 + xy + yz + z^2 = 3$. The equation of the tangent plane at $(1, 2, 0)$ is
  (a) $4x - y - 2z = 6$,
  (b) $4x + y + 2z = 6$,
  (c) $z = 3x - 4y - 10$,
  (d) $3x + 2y + 2z = -6$,
  (e) $z = 3x + 4y - 10$.
  Answer: (b)

(6) Of the two infinite series

$$A : \sum_{n=1}^{\infty} e^{-2n} \sin^4 n \qquad\qquad B : \sum_{n=1}^{\infty} \frac{1}{n+10}\sqrt{1-\frac{1}{n}}$$

(a) both are absolutely convergent
(b) A is absolutely convergent and B is conditionally convergent
(c) A is divergent and B is convergent
(d) A is convergent and B is divergent
(e) both are divergent
Answer: (d)

(7) The coefficient of x^4 in the Maclaurin series for $\dfrac{1}{\cos(x)}$ is
(a) -1, (b) $-1/2$, (c) $\frac{5}{24}$, (d) $1/2$, (e) 1.
Answer: (c)

(8) Let s denote the sum of the alternating series $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{3n+2}$, then
(a) $s < 0$, (b) $0 < s < 2/3$, (c) $\dfrac{2}{3} < s < 7/8$, (d) $7/8 < s < 1$, (e) $s > 1$.
Answer: (a)

(9) The unit tangent $\vec{T}$ to the curve $\vec{R} = \left(\frac{1}{2}t^3, t, t^2\right)$ when $t = 2$ is parallel to
(a) $24\vec{\imath} + 2\vec{\jmath} + 8\vec{k}$
(b) $18\vec{\imath} + \vec{\jmath} + 6\vec{k}$
(c) $24\vec{\imath} + 1\vec{\jmath} + 2\vec{k}$
(d) $12\vec{\imath} + 2\vec{\jmath} + 8\vec{k}$
(e) $12\vec{\imath} + 6\vec{k}$
Answer: (d)

(10) The tangent plane to the surface $z = y\ln(y + 2x - 3)$ at $(1, 2, 0)$ is
(a) $z = 4x + 2y$,
(b) $z = -4x - 2y + 8$,
(c) $z = 4x + 2y - 8$,
(d) $z = 4x + 3y - 10$,
(e) $z = 2x + 2y - 6$.
Answer: (c)

(11) The directional derivative of $z = x\ln(x+2y-3)$ at the point $(2,1)$ in the direction parallel to the vector $(3,4)$ is
(a) 22, (b) -2.8, (c) 2.8, (d) -4.2, (e) 4.4.
Answer: (e)

(12) Let $z(x,y)$ be defined implicitly by $z^3 + z^2y + zx = 5$. Then $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y}$ at the point $(1,3,1)$ has the value
(a) -20, (b) -2, (c) -0.2, (d) 0, (e) 0.2.
Answer: (c)

(13) One of the critical points of $F(x,y) = x^3 + 3y^2 - 6xy$ is at
(a) $(1,1)$, (b) $(2,-1)$, (c) $(0,1)$, (d) $(0,0)$, (e) $(0,2)$.
Answer: (d)

(14) The arc length along the curve $\vec{R} = \left(t^2, \dfrac{t^3}{3}, 2t\right)$ from the origin to its intersection with the plane $z = 2$ is
(a) $\dfrac{7}{5}$, (b) $\dfrac{1}{2}$, (c) $\dfrac{7}{3}$, (d) $\dfrac{2}{3}$, (e) 2.
Answer: (c)

(15) The power series $\sum_{n=1}^{\infty}(-1)^n \dfrac{(x+3)^{3n}}{8^n\sqrt{(n+1)}}$ has interval of convergence
(a) $1 < x < 5$,
(b) $-5 \le x \le -1$,
(c) $0 \le x < 3$,
(d) $-6 < x \le 0$.
(e) $-5 < x \le -1$.
Answer: (e)