MATH222
ASSIGNMENT 6
DUE WEDNESDAY NOV. 7, 2007

1. The function $z = Z(x, y)$ is defined implicitly by the equation

$$x^2 + y^2 + 4z^2 + z^4 = 64$$

   (a) Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial x^2}$, and then evaluate these at the point $P = (4, 4, 2)$.

   (b) Find the equation of the tangent plane to the surface defined above at the point P.

   (c) Find ∇Z at P and find the direction of maximum increase of the function Z.

2. Find the critical points indicating (with your justification) which are local maxima, local minima, and which are saddle points of the function.

$$z = xy\ e^{-(x^2+4y^2)/2}$$

3. Let z be defined implicitly by $z^3 + 4z = 2x^2y + 12$. Taking x and y as independent variables, find all the first and second partials of z and evaluate these at $(x, y) = (1, 2)$ given that $z(1, 2) = 2$.

4. Let $f(x, y) = 4x^2 + 2y^3 - 3xy^2 - 24y + 40$.

   (a) Find and classify all the critical points of the function $f(x, y)$,

   (b) Find the absolute maximum and minimum values of $f(x, y)$ in the region $\{0 \le x \le 2, \quad 0 \le y \le 1\}$.

5. An open rectangular box has volume 128 cubic centimeters. Among all such boxes, what are the dimensions of the box with minimum surface area?

6. Using the method of Lagrange Multipliers, find the maximum and minimum values of the function

$$12x^2 + 8xy + 12y^2 + z^2$$

   if the point (x, y, z) is constrained to lie on the sphere $x^2 + y^2 + z^2 = 8$