MATH 222 ASSIGNMENT 5 (DUE FRIDAY OCT. 26)

(1) Let $f(x,y) = \frac{2xy}{x^2+y^2} \quad (x,y) \neq (0,0), \quad f(0,0) = 0.$
  (a) find $f_x(0,0), f_y(0,0)$, find both $f_x(x,y)$ and $f_y(x,y)$ at $(x,y) \neq (0,0)$
  (b) Although both the partial derivatives $f_x(0,0)$ and $f_y(0,0)$ exist, show that $f(x,y)$ is not continuous at $(0,0)$.
  (c) show that at $(x,y) \neq (0,0)$

$$x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = 0$$

(2) Show that Laplace's equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$ is satisfied by each of the following functions:
  (a) $V = x^3 - 3xy^2$
  (b) $V = e^{-y} \cos x$
  (c) $V = \tan^{-1}\left(\frac{y}{x}\right)$ whereby $(x,y) \neq (0,0)$.

(3) If $w = f(x,y,z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$, find
  (a) ∇w at any point $(x,y,z) \neq (0,0,0)$
  (b) the rate of increase of w at the point $(2,1,2)$ in each of the following directions:
    (i) $(1,2,2)$
    (ii) $(1,4,8)$
  Leave the answers in terms of simple fractions.

(4) For the surface $z = \sqrt{xy}$ defined for $x > 0, \quad y > 0$,
  (a) find the differential dz, and use your result to find the tangent plane at $(x,y) = (9,4)$ and the normal to the surface at this point.
  (b) Show that the straight line $\mathbf{r}(t) = (kt, \frac{t}{k}, t), \quad (k > 0)$ lies on the surface. Now compute $\frac{dz}{dt}$ at any point on the line.
  (c) Find the value of k which makes the straight line pass through the point defined in the first part of this question, and then show also that this line lies completely in the tangent plane at this point.

(5) Let $v = V(x,y)$ and $x = r\cos\theta, \quad y = r\sin\theta$.
  (a) Find $\frac{\partial V}{\partial r}$ and $\frac{\partial V}{\partial \theta}$ in terms of $\frac{\partial V}{\partial x}$ and $\frac{\partial V}{\partial y}$.
  (b) Now show, for $r \neq 0$

$$\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 = \left(\frac{\partial V}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial V}{\partial \theta}\right)^2$$

(6) Let $\phi = \Phi(x,y,z) = x^2 + y^2 + z^4$ and set

$$x = uv + w, \quad y = (u+v+w)^2, \quad z = w.$$

  (a) Find $\frac{\partial \phi}{\partial u}, \quad \frac{\partial \phi}{\partial v}, \quad \frac{\partial \phi}{\partial w}$.
  (b) Approximate ϕ if $u = 1.01, \quad v = 2.01, \quad w = 1.02$ by using differentials and evaluating ϕ for $u = 1, \quad v = 2, \quad w = 1$.