MATH 222 ASSIGNMENT 3

1. Recall that $\sin^{-1} x = \int_0^x \frac{dt}{\sqrt{1-t^2}}$. Use the binomial theorem and this integral to find the Maclaurin expansion of $\sin^{-1} x$.

2. Find a unit vector perpendicular to both $\mathbf{v} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

3. Show that the points $P(-2, 1, 3)$, $Q(2, 3, 0)$ and $R(-6, -1, 6)$ are collinear.

4. The vector $\mathbf{u}$ makes angles α, β, γ with respectively the X–axis, the Y–axis and the Z–axis. Find γ if $\alpha = \frac{2\pi}{3}$ and $\beta = \frac{\pi}{4}$.

5. Show that the angle between the vector joining (a, b, c) and (b, c, a) and the vector joining (a, b, c) to (c, a, b) is equal to $\frac{\pi}{3}$.

6. Find the distance between the skew lines (show that they are indeed skew before starting the rest of the work):

$$\mathbf{v}(t) = \begin{pmatrix} 7t \\ 2+t \\ 4-3t \end{pmatrix} \quad \text{and} \quad \mathbf{w}(s) = \begin{pmatrix} 3-s \\ 5 \\ 6+2s \end{pmatrix}$$

7. Determine the vector parametric equation of the line through the origin and the intersection of the lines (find the co-ordinates of this intersection as well)

$$\mathbf{v}(t) = \begin{pmatrix} 2t+3 \\ -4t \\ t-3 \end{pmatrix} \quad \text{and} \quad \mathbf{w}(s) = \begin{pmatrix} 10s+3 \\ 5s-25 \\ 4-2s \end{pmatrix}$$