Mathematics 222 Calculus III
Assignment 2

1. For the following power series, find

   (a) the radius of convergence

   (b) the interval of convergence, discussing the endpoint convergence when the radius of convergence is finite.

   (i) $\sum_1^\infty \frac{(x-1)^n}{3n\sqrt{n}}$,

   (ii) $\sum_1^\infty \left(1-\frac{1}{n}\right)^{n^2} x^n$,

   (iii) $\frac{\sum_2^\infty (-1)^n (x+1)^{2n}}{4^n n^2 \log(n)}$

   (iv) $\sum \frac{4^n x^n}{[\log(n+1)]^n}$,

   (v) $J_0(x) = \sum_0^\infty \frac{(-1)^n}{(n!)^2}\left(\frac{x}{2}\right)^n$

2. Given

$$f(x) = \sum_1^\infty \frac{(-1)^{n+1}(x-5)^n}{n5^n}$$

   find the interval of convergence of the Taylor series expansions around $x = 5$ of the following

   (a) $f(x)$, (b) $f'(x)$, (c) $\int_5^x (f(t)dt$

3. If $f(x) = \int_0^x \frac{1-e^{-t}}{t} dt$

   (a) find a power series for $f(x)$ about $x = 0$

   (b) find the interval of convergence of this series.

   (c) compute $f(0.4)$ to four decimal place accuracy justifying your answer

4. (a) Obtain the Taylor series for $f(x) = \frac{3}{x^2-x-2}$ about $x = 1$.

   (b) find the interval of convergence of this series.

   (c) use the series to compute $f^{(6)}(1)$

5. find the first three non-zero terms of the Maclaurin expansion of $y = f(x)$ where the function is defined implicitly by $x^2 + xy + y^2 = 1$. Also estimate the error approximating f(0.1) using the first two non–zero terms of this series.