Math 222: Assignment 1

Problem 1 (2 marks)

Does the sequence $b_n = sin(\frac{\pi}{2} + \frac{1}{n})$ converge or diverge. If it converges what is the limit?

$$\lim_{n\to\infty} sin(\frac{\pi}{2} + \frac{1}{n}) = sin(\frac{\pi}{2} + \lim_{n\to\infty} \frac{1}{n}) = sin(\frac{\pi}{2}) = 1$$

Problem 3 (2 marks)

Does the sequence $b_n = n^2(1 - cos(\frac{1}{n}))$ converge or diverge. If it converges what is the limit?

$$\lim_{n\to\infty} n^2(1 - cos(\frac{1}{n})) = \lim_{n\to\infty} \frac{(1 - cos(\frac{1}{n}))}{\frac{1}{n^2}}$$

Applying L'Hopitals Rule twice we get

$$\lim_{n\to\infty} \frac{(1 - cos(\frac{1}{n}))}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{sin(\frac{1}{n})}{2} = \lim_{n\to\infty} \frac{1}{2} cos(\frac{1}{n})$$

So we get the limit of b_n to be $\frac{1}{2}$.

For the following series, determine whether they converge (absolutely or conditionally) or diverge. Justify your answer.

Problem 5 (4 marks)

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$$

First we note that

$$\sum_{n=1}^{\infty} \mid (-1)^{n+1} \frac{1+n}{n^2} \mid \leq \sum_{n=1}^{\infty} \frac{1}{n} + \sum_{n=1}^{\infty} \frac{1}{n^2}$$

And so is divergent because the harmonic series diverges. So for conditional convergence we notice that $b_{n+1} < b_n$ and that $\lim_{n\to\infty} |b_n| = 0$. So we have conditional convergence.

Problem 7 (4 marks)

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n+1}+\sqrt{n}}$$

We do not have absolute convergence since

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}=\sum_{n=1}^{\infty}\sqrt{n+1}-\sqrt{n}$$

This is a telescoping sum of which the limit is

$$\lim_{n\to\infty}\sqrt{n+1}-1=\infty$$

This series, however is conditionally convergent since $b_{n+1}<b_n$ and that $\lim_{n\to\infty}|b_n|=0$.

Problem 9 (4 marks)

$$\sum_{n=1}^{\infty}\frac{sin(n)\sqrt{n}}{n^2+1}$$

This series is absolutely convergent (and therefore conditionally convergent) since

$$\sum_{n=1}^{\infty}\mid\frac{sin(n)\sqrt{n}}{n^2+1}\mid\leq\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2}=\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$$

This series is convergent from the p-series test.

Problem 11 (4 marks)

$$\sum_{n=1}^{\infty}\frac{sin(\frac{1}{n})}{(ln(1+n))^2}$$

We can see that this is convergent by using the comparison test using the following convergent series

$$\sum_{n=1}^{\infty}\mid\frac{sin(\frac{1}{n})}{(ln(1+n))^2}\mid\leq\sum_{n=1}^{\infty}\frac{1}{n(ln(n))^2}$$

This series can be shown to be convergent by the integral test

$$\int_2^{\infty}\frac{1}{x(ln(x))^2}dx=\frac{1}{ln(x)}\bigg|_{\infty}^{2}=\frac{1}{ln(2}<\infty$$

Therefore the series is conditionally convergent