

Moment zeta functions in arithmetic geometry

Daqing Wan (dwan@math.uci.edu)

University of California at Irvine

Department of Mathematics

Irvine, CA 92687-3875

USA

Abstract. The Riemann zeta function counts the number of primes (closed points) of the ring of integers. More generally, the Hasse-Weil zeta function counts the number of closed points of a scheme X of finite type over the integers. To understand the distribution of the closed points on X , we introduce a k -th moment zeta function of X for each positive integer k , which reduces to the classical Hasse-Weil zeta function in the case $k = 1$. In this lecture, we give a simple and expository introduction to the possible meromorphic continuation and Riemann hypothesis for these higher moment zeta functions, with some emphasis on the geometric case in connection to the conjectures of Weil and Dwork. The arithmetic limiting behavior of the k -th moment zeta function when k goes to infinity seems to be especially interesting, leading to a possible limiting moment zeta function.

