Torsion points on curves and common divisors of $a^k - 1$ and $b^k - 1$

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Abstract. We study the behaviour of the greatest common divisor of $a^k - 1$ and $b^k - 1$, where a, b are fixed integers and k varies. We conjecture that when a and b are multiplicatively independent and in addition $a - 1$ and $b - 1$ are coprime, then $a^k - 1$ and $b^k - 1$ are coprime infinitely often. A strong version of this conjecture can be proved in the function field case, as a consequence of a result of Lang's on the finiteness of torsion points on algebraic curves. We will also discuss an elliptic analogue of these results. (joint work with Nir Ailon).