Bertini theorems over finite fields

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Abstract. One form of Bertini's theorem states that if X is a smooth projective variety of dimension m in P^n over an infinite field k, then there exists a hyperplane H defined over k such that the intersection of X and H is smooth of dimension $m-1$. This can fail if k is finite. Katz asked whether the statement would remain true if "hyperplane" were changed to "hypersurface". We give an affirmative answer. In fact, as d tends to infinity, the fraction of hypersurfaces of degree d that are good tends to a special value of the zeta function of X. Sketch of proof: sieve out the bad hypersurfaces and count carefully to show that something remains...

A generalization of our result answers another question of Katz, about "space filling curves": if X is a smooth projective variety of dimension $m > 1$ over a finite field k, does there exist a smooth projective curve Y over k in X with $Y(k) = X(k)$?