The structure of fundamental groups of number fields

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Abstract. Given a number field K, a set S of places of K, and a prime number p, we consider the Galois group of the maximal pro-p-extension of K unramified outside S (this is the fundamental group referred to in the title). It is finitely generated as a pro-p-group, with a finite number of relations, and the number of generators and relations can be calculated in terms of the Galois cohomology of K. Beyond these basic facts, the structure of the fundamental group is quite mysterious. We survey recent developments, including work on Greenberg's conjecture (McCallum, Marshall, Nguyen-Quang-Do, and Lannuzel) and some calculations with Romyar Sharifi on the structure of the relations.