Inequities in the Shanks-Rényi prime number race

Greg Martin (gerg@math.ubc.ca)
University of British Columbia
Department of Mathematics
Room 121, 1984 Mathematics Road
Vancouver, BC V6T 1Z2
Canada

Abstract. It has been well-observed that an inequality of the type $\pi(x;q,a) > \pi(x;q,b)$ is more likely to hold if a is a non-square modulo q and b is a square modulo q (the so-called "Chebyshev Bias" in comparative prime number theory). However, it has come to light that the tendencies of the various $\pi(x;q,a)$ (for nonsquares a) to dominate $\pi(x;q,b)$ have different strengths. A related phenomenon is that the six possible inequalities of the form $\pi(x;q,a1) > \pi(x;q,a2) > \pi(x;q,a3)$, with $a1$, $a2$, $a3$ all non-squares modulo q, are not all equally likely; some orderings are preferred over others. For given values $q, a, b, \ldots$, these tendencies can be quantified and computed, but only using laborious numerical integration of functions involving zeros of the appropriate Dirichlet L-functions. In this talk we present a framework for explaining which nonsquares a are most dominant for a given square b, for example, based only on elementary properties of the congruence classes a modulo q rather than the complicated computations just mentioned.