

Plenary Lecture, Room E-310
Friday, May 24, 9:00-9:45

Wavelets, tilings and number theory

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Abstract. This talk considers orthonormal wavelet bases of the Hilbert space of square-summable functions on n -dimensional Euclidean space. These are orthonormal bases formed by translates and dilations of a single function; the Haar basis is the prototype example. Such wavelets are specified by a scaling function, which is a solution of a functional difference equation, called a dilation equation. This equation involves a dilation map which takes x to Mx , where M is an integer n by n matrix which is expanding in the sense that all its eigenvalues are of length exceeding one. Ingrid Daubechies showed there exist orthonormal bases of compactly supported wavelets of arbitrary smoothness for dilations taking x to $2x$ on the line. Do such wavelets exist for all dilation matrices M ? We consider Haar-type wavelets. Their existence is related to radix expansions to base M having nice properties, and lead to various problems in number theory, some solved and some unsolved.

