

# Applications of the functorial symmetric cube and symmetric fourth

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**Abstract.** We give several applications of the functorial symmetric cube and symmetric fourth of cuspidal representations of  $GL(2)$ . First, we establish the bound  $q_v^{1/9}$  for unramified Hecke eigenvalues of cusp forms on  $GL(2)$  over an arbitrary number field. Over  $\mathbb{Q}$ , the bound can be improved slightly to  $7/64$ . Second, we prove that given a cuspidal representation of  $GL(2)$ , the set of tempered places has lower Dirichlet density of at least  $34/35$ . Third, following Serre's method on Sato-Tate conjecture, given a cuspidal representation  $\pi$  of  $GL(2)$  with the trivial central character, we prove that there exists a set  $T$  of positive lower density such that  $|a_v| > 1.68 \dots$  for all  $v \in T$ , where  $a_v$  is the (normalized) Fourier coefficient. This is a joint work with F. Shahidi.

