Modularity for Drinfeld modules

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Abstract. One of the great triumphs of 20th Century mathematics was the proof (by C. Breuil, B. Conrad, F. Diamond, and R. Taylor following Andrew Wiles) that all elliptic curves over $\mathbb{Q}$ are modular; that is, such elliptic curves are isogenous to factors of Jacobians of classical elliptic modular curves.

The modularity of elliptic curves also holds in characteristic p. Indeed, due to the work of Drinfeld and Zarhin, elliptic curves over $Fq(T)$, with the appropriate reduction at the infinite prime, are modular in that they are isogenous to factors of the Jacobian of modular curves parameterizing Drinfeld modules of rank 2.

But what about the Drinfeld modules themselves? These are affine objects in that the underlying space is the additive group. Can they also be modular? In this talk we will discuss some of the results of Gebhard Boeckle in his 2002 ETH habilitation thesis which allow us to present a first reasonable statement along these lines.

(Boeckle's thesis is entitled: "An Eichler-Shimura isomorphism over function fields between Drinfeld modular forms and cohomology classes of crystals" and may be found on http://www.math.ethz.ch/ boeckle/)