Multivariate Bateman-Horn conjecture

Brian Conrad (bdconrad@math.sa.umich.edu)
University of Michigan
Department of Mathematics
525 East University Ave.
Ann Arbor, MI 48109-1109
USA

Abstract. A classical conjecture of Bateman and Horn predicts asymptotics on how often a finite set of one variable polynomials over $\mathbb{Z}$ simultaneously take on prime values. Since the special case T, $T+2$ is the twin prime conjecture (with an asymptotic!), chances of a proof in the near future are remote. Nevertheless, one can ask for more: what about polynomials in several variables over S-integers of global fields?

We discuss the heuristics in this case, state the conjecture, and provide some compelling evidence (both theoretical and numerical). The proof of convergence of the leading coefficient in the asymptotic rests on Weil II. This is joint work with Keith Conrad and Robert Gross.