Counting discriminants of number fields

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Abstract. In this talk, we will explain several modern methods which have been used to count, both exactly and asymptotically, and both in the absolute and the relative case, number fields having a given Galois group (of Galois closure) with discriminant up to a given bound. Even though the abelian case is in principle not difficult, the relative case entails considerable difficulties and the use of classical but subtle techniques of algebraic number theory. In this manner, we solve the cyclic case of prime order, the cyclic case of order 4 and the case of the Klein 4-group. In the nonabelian case, we solve for the first time, both in the absolute and relative situation the case of the dihedral group of order 8. The exact computation of the number of such number fields involves in a very pretty manner Gauss's classical theory of genus characters, and when signature conditions are added we must also study interesting non genus characters. Finally, for the alternating and symmetric group of degree 4, we find exact formulas which enable us to compute large systematic tables (both of the fields themselves and of their number), but not to find asymptotic formulas. Finally, we will mention very general conjectures of G. Malle, as well as the work of M. Bhargava which solves the problem in the absolute case for the symmetric group of degree 4.