On approximation of real, complex, and p-adic numbers by algebraic numbers

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Abstract. The problem of approximating by algebraic numbers is of classical interest in the theory of Diophantine approximation. In 1842 Dirichlet proved that for any real irrational number ξ there exist infinitely many rational numbers p/q such that $|\xi - p/q| < q^{-2}$. Let $\mathbb{A}_n$ be the set of algebraic numbers of degree $\leq n$. It is very natural to suppose, that Dirichlet's Theorem can be generalized to the case of approximation by algebraic numbers $\alpha \in \mathbb{A}_n,\ n > 1$. In 1961 E. Wirsing conjectured that for any real number $\xi \notin \mathbb{A}_n$ and any $\epsilon > 0$ there exist infinitely many algebraic numbers $\alpha \in \mathbb{A}_n$ with $|\xi - \alpha| \ll H(\alpha)^{-n-1+\epsilon}$, where $H(\alpha)$ is the height of α. Further W. M. Schmidt conjectured that the exponent $-n-1+\epsilon$ can be replaced by $-n-1$. This problem has not been solved except in some special cases. The talk will discuss major results, applied methods, and other unsolved problems.