Fibre product of Artin-Schreier and Kummer extensions and curves with many points

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Abstract. Let F_q be a finite field of cardinality $q = p^n$ with p a prime number. We study algebraic function fields over F_q (or equivalently, algebraic curves over F_q) given by generators $z_1, \ldots, z_r, y_1, \ldots, y_s$ and relations of the form

$$y_i^p - y_i = f_i(x) \quad z_i^n = g_j(x)$$

where n is a divisor of $q - 1$.

Under some suitable conditions on $f_i(x)$ and $g_j(x)$, the system of equations defines a curve in the space A^{s+r+1}.

Denote by $\mathrm{Tr}_{q/p}$ and $N_{q/p}$ the trace map from F_q to F_p and the norm map, respectively.

If one chooses the functions $f_i(x)$ and $g_j(x)$ to be in the kernel of $\mathrm{Tr}_{q/p}$ and $N_{q/p}$ respectively, for a large set in F_q, then one gets algebraic curves defined over F_q with many F_q-rational points.

We compute the genus of the curve defined by this system and we construct some polynomials $f_i(x)$ and $g_j(x)$ such that the system of equations has many rational points.