Computing all S-integral solution in a family oftwo simultaneous Pell equations

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Abstract. Consider the two Pell equations

$$x^2 + d1y^2 = a1 \quad \text{and} \quad z^2 + d2y^2 = a2 \tag{1}$$

where $a1d2 - a2d1$ is non-zero for fixed integers $a1$, $a2$, $d1$, $d2$. Denote by S a finite set of primes which includes the prime at infinity. To compute all simultaneous S-integer solutions of these equations an explicit transformation to an elliptic curve E will be described. It will be shown that every S-integral solution of (1) will map to an S-integral solution of E. To compute S-integral solutions on E you may use the method of complex and p-adic elliptic logarithms. This method will be discussed briefly. Finally, some computational results will be given.