

Vanishing of twisted L -functions of an elliptic curve and rational points on K^3 surfaces

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Abstract.

Let E be an elliptic curve defined over the rational number field. We are interested in how the rank of the group of K -rational points $E(K)$ varies when K runs through a certain family of field extensions. Particularly interested is the case where K runs through all cyclic extensions of fixed degree. Though a generalization of the conjecture of Birch and Swinnerton-Dyer, our problem amounts to study the vanishing of the L -function of E twisted by Dirichlet characters of fixed order.

When the degree or extension is 3 or 4, we show that the existence of a K -rational point on E translates to the existence of a rational point of a certain K^3 surface. In the degree 4 case we show that the K^3 surface in question has always infinitely many rational points, and thus there are infinitely many cyclic extensions K of degree 4 such that the rank of $E(K)$ is positive.

