On the divisor problem: Moments of $\Delta(x)$ over short intervals

Werner Georg Nowak (`nowak@mail.boku.ac.at`)
Universitaet fuer Bodenkultur
Institut fuer Mathematik u. Ang. Stat.
Peter Jordan Strasse 82
A-1190 Vienna
Austria

Abstract. Let as usual $\Delta(x)$ denote the error term in the Dirichlet divisor problem, i.e.,

$$\sum_{n \le x} d(n) = x \log x + (2\gamma - 1)x + \Delta(x),$$

x a large real variable and γ the Euler-Mascheroni constant. While asymptotics for the square-mean of $\Delta(x)$ are classic, K.-M. Tsang [Proc. London 1992] recently established results on the third and fourth power moments.

We present short interval variants of such asymptotics.

THEOREM 1. *For T a large real variable, suppose that $T \mapsto \Lambda = \Lambda(T)$ increases with T, satisfies $0 < \Lambda(T) \le \frac{1}{2}T$, and*

$$\lim_{T \to \infty} \frac{(\log T)^3}{\Lambda(T)} = 0.$$

Then, as $T \to \infty$,

$$\int_{T-\Lambda}^{T+\Lambda} \left(\Delta(t^2)\right)^2 \mathrm{d}t \sim \frac{1}{2\pi^2} \frac{\zeta^4(3/2)}{\zeta(3)} \Lambda T.$$

THEOREM 2. *For T, $\Lambda(T)$ as above, suppose in addition that $\lim\limits_{T \to \infty} \Lambda(T)/T =: \lambda$ exists, and, for some $\epsilon_0 > 0$,*

$$\lim_{T \to \infty} \frac{T^{1/2+\epsilon_0}}{\Lambda(T)} = 0.$$

Then, as $T \to \infty$,

$$\int_{T-\Lambda}^{T+\Lambda} \left(\Delta(t^2)\right)^3 \mathrm{d}t \sim C_3(\lambda) \Lambda T^{3/2},$$

and

$$\int_{T-\Lambda}^{T+\Lambda} \left(\Delta(t^2)\right)^4 \mathrm{d}t \sim C_4(\lambda) \Lambda T^2,$$

with explicite positive coefficients $C_3(\lambda), C_4(\lambda)$.