Variance of distribution of almost primes in arithmetic progressions

Emmanuel Knafo (knafo@math.toronto.edu)
University of Toronto
Department of Mathematics
Toronto, ON M5S 3G3
Canada

Abstract. In counting primes up to x in a given arithmetic progression, one resorts to the 'prime' counting function

$$\psi(x;q,a) = \sum_{\substack{n\leq x \\ n\equiv a \pmod q}} \Lambda(n)$$

where Λ is the usual von Mangoldt function. Analogously, to count those integers with no more than k prime factors, one can use

$$\psi_k(x;q,a) = \sum_{\substack{n\leq x \\ n\equiv a \pmod q}} \Lambda_k(n)$$

where Λ_k is the generalized von Mangoldt function defined by $\Lambda_k = \mu * \log^k$. Friedlander and Goldston gave a lower bound of the correct order of magnitude for the mean square sum

$$\sum_{\substack{a \pmod q \\ (a,q)=1}} \left(\psi(x;q,a) - \frac{x}{\phi(q)}\right)^2$$

for q in the range $\frac{x}{(\log x)^A} \leq q \leq x$. Later, Hooley extended this range to $\frac{x}{\exp(c\sqrt{\log x})} \leq q \leq x$. We obtain, in the larger range, a lower bound of the correct order of magnitude and approaching the expected asymptotic as k approaches infinity.