Nonvanishing results for L-functions associated to quadratic twists of Newforms

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Abstract. This work focuses on relating the Hecke structure of spaces of cusp forms of half-integral weight to the nonvanishing of central critical values of L-functions associated to integral weight newforms. By determining the multiplicities of distinct Hecke eigenforms with the same Hecke eigenvalues, in prior work I fully "decompose" a space of half-integral weight cusp forms and characterize its Hecke structure. These decompositions are given in terms of conditions on the primes dividing the level of the forms. They give a classical reformulation of results of Flicker and Waldspurger, determining when an integral weight newform will have equivalent half-integral weight cusp forms of a given level and character. There is a connection between these Hecke structure results and the nonvanishing of central critical values of L-functions associated to quadratic twists of newforms. This connection is made via work of Waldspurger relating these central critical values to Fourier coefficients of half-integral weight cusp forms. Through work of Ono & Skinner, among others, my Hecke structure results generate a lower bound on the occurrence of the first quadratic twist with nonvanishing central critical value. In addition, I conjecturally give a corresponding "positive proportion" result regarding nonvanishing of central critical values of quadratic twists with a specified (small) number of prime factors.