Normal integral bases in quadratic and cyclic cubic extensions of quadratic fields

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Abstract. Let K be a number field and let G be a finite abelian group. We call K a Hilbert-Speiser field of type G if, and only if, every tamely ramified normal extension L/K with Galois group isomorphic to G has a normal integral basis. Now let C_2 and C_3 denote the cyclic groups of order 2 and 3, respectively. Firstly, we show that among all imaginary quadratic fields, there are exactly 3 Hilbert-Speiser fields of type C_2: $\mathbb{Q}(\sqrt{m})$, where $m \in \{-1, -3, -7\}$. Secondly, we give some necessary and sufficient conditions for a real quadratic field $K = \mathbb{Q}(\sqrt{m})$ to be a Hilbert-Speiser field of type C_2. These conditions are in terms of the congruence class of m modulo 4 or 8, the fundamental unit of K, and the class number of K. Finally, we show that among all quadratic number fields, there are exactly 8 Hilbert-Speiser fields of type C_3: $\mathbb{Q}(\sqrt{m})$, where $m \in \{-11, -3, -2, 2, 5, 17, 41, 89\}$.