Remarks on Assignment 2

November 11, 2009

I have noticed some trouble in proving a lemma which is a hint for exercise 33. Here is the proof:

Lemma. *Let Q be an abelian group, and let $q \in Q$ be a nonzero element. Then there exists $h\colon Q \to \mathbb{Q}/\mathbb{Z}$ such that $h(q) \neq 0$.*

Proof. Consider the subgroup $\langle q \rangle \subseteq Q$, generated by the element q. It is a cyclic abelian group, and thus isomorphic to either $\mathbb{Z}$ or to $\mathbb{Z}/m\mathbb{Z}$ for some $m \in \mathbb{Z}$. In the first case, define $\tilde{h}\colon \langle q \rangle \to \mathbb{Q}/\mathbb{Z}$ by $\tilde{h}(tq) := \frac{t}{2} \mod \mathbb{Z}$. In the second case, set $\tilde{h}(tq) := \frac{t}{m} \mod \mathbb{Z}$. This is a group homomorphism, and since $\mathbb{Q}/\mathbb{Z}$ is injective (over $\mathbb{Z}$) it can be extended to $h\colon Q \to \mathbb{Q}/\mathbb{Z}$. But $h(q) = h(\iota(q)) = \tilde{h}(q) \neq 0$, as wanted.

$$\begin{tikzcd} 0 \arrow[r] & \langle q \rangle \arrow[r, "\iota"] \arrow[d, "\tilde{h}"'] & Q \arrow[dl, dashed, "h"] \\ & \mathbb{Q}/\mathbb{Z}. & \end{tikzcd}$$

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