TOPICS IN ALGEBRAIC GEOMETRY, MATH 722, FALL 2012
- SYLLABUS -(*)

EYAL GOREN

(1) Orientation meeting:
   (a) What is algebraic geometry?
   (b) Prerequisites and administrative aspects.
(2) Algebraic sets and affine varieties.
   (a) Algebraic sets.
   (b) Some topology.
   (c) The fundamental theorem of affine varieties.
   (d) Affine and quasi-affine varieties and coordinate rings.
   (e) Dimension and height.
(3) Projective varieties.
   (a) Graded rings.
   (b) Conical sets.
   (c) Projective space.
   (d) Algebraic sets in $\mathbb{P}^n$.
   (e) The Grassmann variety.
(4) Regular functions and morphisms
   (a) Sheaves.
   (b) The sheaf of regular functions.
   (c) Morphisms.
(5) Products.
   (a) Products of affine varieties.
   (b) Products of general varieties.
   (c) Application to morphisms.
(6) Rational functions and morphisms.
   (a) The field of rational functions.
   (b) Birational equivalence.
(7) Singularities.
   (a) Tangent space.
   (b) Singular points and the singular locus.
   (c) Tangent cone.
   (d) The completion of the local ring.
(8) Blow-up.
   (a) Definition of blow-up at a point.
   (b) Examples.
   (c) Rephrasing the definition.
   (d) Blow-up along a subvariety.
   (e) Deformation to the normal cone.
   (f) Examples and a taste of intersection theory.
(9) Grobner basis and computing in algebraic geometry.

(*) *FOR GENERAL ADMINISTRATIVE ASPECTS, INCLUDING SCHEDULE, EVALUATION SCHEME AND TEXT BOOKS RECOMMENDATIONS, SEE THE COURSE WEB-PAGE TO WHICH YOU CAN NAVIGATE FROM WWW.MATH.MCGILL.CA/GOREN*

   (a) The membership problem.
   (b) Orders on monomials.
   (c) Grobner bases.
   (d) Buchberger's algorithm.
   (e) Calculating the projective closure and other applications.
(10) Integrality and finite morphisms.
   (a) Definitions.
   (b) The main theorems and their geometric content.
   (c) Noether's normalization lemma.
(11) Toric varieties.
   (a) Fans and cones.
   (b) Toric varieties.
   (c) Examples.
   (d) A criterion for regularity and normality.
(12) Curves.
   (a) Discrete valuation rings.
   (b) Curves.
   (c) The equivalence between curves and function fields of transcendence degree 1.
(13) Intersections in projective space.
   (a) Introduction: Bezout's theorem.
   (b) The Hilbert polynomial.
   (c) Intersection theory in $\mathbb{P}^n$.
(14) Further examples and applications.
   (a) Projection from a point.
   (b) Dimension and its behaviour under a morphism.