ASSIGNMENT 7 - MATH576, 2006

(This is the last assignment!!)

Solve the following questions. Submit by Monday, December 4.

(1) Do Exercise 4, page 353.
(You may omit the proof of (d) and (e), since we did those in class. Refer to the textbook for the definition of a vector field.)

(2) Do Exercise 10, page 367.
(Part (d) of this exercise for $n = 2$ is called the *hairy ball theorem*. It can be phrased as saying that it is impossible to comb a hairy ball (the hairs then will give a non-vanishing tangent vector field). It also says that at any given moment, somewhere on earth there is a place where the wind doesn't blow.)

(3) Let X be a bouquet of n circles.
  (a) How many pointed covering spaces of degree 2 does X have? How many of which are regular?
  (b) How many pointed covering spaces of degree 3 does X have? How many of which are regular?

(4) Find a topological space whose fundamental group is $\langle x, y : x^2, y^2 \rangle$.

(5) (a) Find a triangulation of the real projective plane $\mathbb{RP}^2$ and use it to calculate its homology groups.
  (b) Next, using any kind of homology and the list of properties we mentioned, calculate the homology of $\mathbb{RP}^2$ via the Mayer-Vietoris sequence, interpreting $\mathbb{RP}^2$ as the quotient of the upper hemisphere of S^2 with identifications $x \leftrightarrow -x$ along the equator.