Solve the following questions. Submit by Monday, October 2.

(1) Let $X$ be a compact topological space and $f : X \rightarrow \mathbb{R}$ a continuous map. Prove that $f$ has a maximum and minimum.

(2) Do Exercise 5, page 171.

(3) Do Exercise 7, page 171.

(4) Do Exercise 8, page 171. Also, show that neither assumption on $Y$ can be removed.


(6) (Bonus Question.)\footnote{That doesn’t mean it’s necessarily harder than the other questions.} A topological space $X$ has the \textit{fixed point property} if any continuous map $f : X \rightarrow X$ has a fixed point.

(a) Let $X = [0,1]$. Prove that $X$ has the fixed point property (you can use analysis). How about $\mathbb{Q} \cap [0,1]$?

(b) Let $X = \{0\} \times [-1,1] \cup \{(x, \sin(1/x)) : x \in (0,1]\}$. Prove that $X$ is compact. $X$ is called the closed \textit{topologist’s sine curve}. Does $X$ have the fixed point property?