September 16, 2006

ASSIGNMENT 3 - MATH576, 2006

Solve the following questions. Submit by Monday, October 2.

(1) Let X be a compact topological space and $f : X \to \mathbb{R}$ a continuous map. Prove that f has a maximum and minimum.

(2) Do Exercise 5, page 171.

(3) Do Exercise 7, page 171.

(4) Do Exercise 8, page 171. Also, show that neither assumption on Y can be removed.

(5) Do Exercise 7, page 181.

(6) (**Bonus Question.**)[1] A topological space X has the *fixed point property* if any continuous map $f : X \to X$ has a fixed point.

  (a) Let $X = [0, 1]$. Prove that X has the fixed point property (you can use analysis). How about $\mathbb{Q} \cap [0, 1]$?

  (b) Let $X = \{0\} \times [-1, 1] \cup \{(x, \sin(1/x)) : x \in (0, 1]\}$. Prove that X is compact. X is called the closed *topologist's sine curve.* Does X have the fixed point property?

[1] That doesn't mean it's necessarily harder than the other questions.