September 10, 2006

ASSIGNMENT 1 - MATH576, 2006

Solve the following questions. Submit by Monday, September 18.

(1) Do exercise 8 on page 83.

(2) Do exercise 1 on page 91.

(3) Do exercises 6, 7, 8 on page 101.

(4) Do exercise 19 on page 102.

(5) For the following sets S find $\overline{S}, S^0$ and ∂S in $\mathbb{R}_\ell$:

(i) $(0,1)$; (ii) [0, 1); (iii) $(0,1]$.

(6) Supply the details in Furstenberg's proof of the infinitude of primes. At this point ignore the sentence "In fact, under this topology, S may be shown to be normal and hence metrizable."

(7) Do your best regarding question 21 on page 102. (It's good gymnastics, but totally non-important fact).

(8) A topological space X is called T_0 if for any two distinct points a, b in X there is an open set U such that either $a \in U, b \notin U$ or $b \in U, a \notin U$. It is called T_1 if there are open sets U_a, U_b such that $a \in U_a, b \notin U_a$ and $b \in U_b, a \notin U_b$. It is called T_2, or Hausdorff, if the open sets U_a and U_b can be chosen to be disjoint. Those properties are called separation axioms and there are many others (we'll encounter the separation axioms "regular" and "normal" later on). Obviously, $T_2 \Rightarrow T_1 \Rightarrow T_0$. Show that $T_0 \not\Rightarrow T_1$, $T_1 \not\Rightarrow T_2$. Show also that T_1 is equivalent to saying that every point is a closed set.