Midterm Exam in MATH 576 (Geometry and Topology I)

October 25, 2006

Answer at least 4 questions for full marks. The use of books or notes is not allowed.

1. Prove that a compact Hausdorff space is normal.

2. Let X be a topological space and $A \subset X$ a dense subset, i.e., $\overline{A} = X$. Let Y be a Hausdorff space and $f, g : X \to Y$ two continuous maps that agree on A. Prove that $f = g$.

3. Let X be a compact Hausdorff space.
   (1) Prove that X is metrizable if and only if X has a countable basis.
   (2) Give an example of a compact Hausdorff space which is not metrizable.

4. Recall that $\mathbb{R}_\ell$ is $\mathbb{R}$ with the topology having as basis half open intervals of the form $[a, b)$, $a < b$. Show that every continuous function $f : \mathbb{R} \to \mathbb{R}_\ell$ (where $\mathbb{R}$ on the left hand side is given the usual topology) is constant. (Suggestion: determine the connected components of $\mathbb{R}_\ell$ first).

5. Let $f : S^1 \to \mathbb{R}$ be a continuous map. Show that there is a point $x \in S^1$ such that $f(x) = f(-x)$.