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ON THE INFINITUDE OF PRIMES

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In this note we would like to offer an elementary "topological" proof of the infinitude of the prime numbers. We introduce a topology into the space of integers S, by using the arithmetic progressions (from $-\infty$ to $+\infty$) as a basis. It is not difficult to verify that this actually yields a topological space. In fact, under this topology, S may be shown to be normal and hence metrizable. Each arithmetic progression is closed as well as open, since its complement is the union of other arithmetic progressions (having the same difference). As a result, the union of any finite number of arithmetic progressions is closed. Consider now the set $A=\bigcup A_p$, where A_p consists of all multiples of p, and p runs through the set of primes $\geqq 2$. The only numbers not belonging to A are -1 and 1, and since the set $\{-1, 1\}$ is clearly not an open set, A cannot be closed. Hence A is not a finite union of closed sets which proves that there are an infinity of primes.

A STATISTICAL DERIVATION OF A PAIR OF TRIGONOMETRIC INEQUALITIES

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The following inequalities and a particular generalization of them can be obtained by comparing the variances of a pair of minimum variance estimators with the corresponding variances of certain less efficient estimators:† Given $\theta=(\theta_1, \theta_2, \cdots, \theta_n)$ such that $0\leqq\theta_i<\pi$ and $\theta_j\neq\theta_k$ for $j\neq k$. Then

$$(1)\quad \frac{\sum_{i=1}^{n}\cos^2\theta_i}{\sum\sum_{j>i}\sin^2(\theta_i-\theta_j)} \leqq \binom{n}{2}^{-2}\sum_{i=1}^{n}\left\{\frac{\cos\theta_1}{\sin(\theta_i-\theta_1)}+\cdots+\frac{\cos\theta_{i-1}}{\sin(\theta_i-\theta_{i-1})}+\frac{\cos\theta_{i+1}}{\sin(\theta_i-\theta_{i+1})}+\cdots+\frac{\cos\theta_n}{\sin(\theta_i-\theta_n)}\right\}^2$$

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† So far as the author has been able to determine, the inequalities stated in this paper appear to be new. The statistics involved can be considered as respective pairs of estimators of the coordinates of a fixed point in a plane, based on line-of-sight observations. The minimum variance estimators (under the conditions stated) are the coordinates of the point from which the sum of squares of the distances to each line-of-sight is a minimum. The less efficient estimators correspond to the estimate (of the location of the fixed point) obtained by averaging arithmetically the vectors determined by the intersections of all pairs of lines of sight.