COURSE SYLLABUS FOR MATH 570/571, 2011

EYAL GOREN

Fall term

(1) Groups – Selected topics.
   (a) Group actions on sets. (Definition, reformulation as $G \to \Sigma_S$, orbits, stabilizers, Cauchy-Frobenius Formula, revisiting Lagrange's theorem, the coset representation....)
   (b) The class equation, Sylow's theorems, and some basic results on p-groups. Applications (groups of order pq, pq^2, solvability of groups of order less than 60).
   (c) Simplicity of A_n and $\mathrm{PSL}_n(\mathbb{F}_q)$.
   (d) Solvable groups.
   (e) Free groups and free products of groups and the concepts of category, functor, adjoint functors. Universal property in terms of initial/final object.

(2) Modules, Part I.
   (a) A recall of the basic definitions and theorems for modules.
   (b) Modules over PID and applications.
   (c) Localization.
   (d) Free modules.

(3) Categories.
   (a) Recall: The language of categories, functors and universal objects.
   (b) Equivalence of categories.
   (c) Injective and projective limits.

(4) Fields.
   (a) Recall: The fundamental theorem of Galois theory and some basic results in Galois theory.
   (b) Profinite groups and, in particular, $\mathbb{Z}_p$.
   (c) Infinite Galois theory.
   (d) Finite fields and cyclotomic fields.
   (e) Kummer theory.
   (f) Solvability by radicals.
   (g) Calculation of Galois groups.

(5) Rings, Part I.
   (a) The spectrum of a ring.
   (b) Integral extensions and the going-up and going-down theorems.

Winter term

(5) Rings, Part I.
   (a) Recall and complements: The spectrum of a ring. (added: the sheaf of regular functions, morphisms.)

(b) Recall and complements: Integral extensions and the going-up and going-down theorems. (added: more examples: $\mathbb{Z}[\sqrt{2},\sqrt{3}]$, $y^2=x^3, y^2=x^2(x+1)$)
(c) Noether's normalization lemma, Hilbert's Nullstellensatz (weak form).
(d) Noetherian and Artinian rings.
(e) Hilbert's basis theorem. Hilbert's Nullstellensatz (strong form).
(f) Dedekind rings: discrete valuation rings and local Dedekind domains. Global Dedekind domains. Unique factorization into ideals in Dedekind domains.

(6) Modules, Part II.
(a) Tensor products.
(b) Exactness properties of Hom and $\otimes$.
(c) Projective modules.
(d) Injective modules.
(e) Flat modules.

(7) Rings, Part II.
(a) The Jacobon radical.
(b) Nakayam's lemma.
(c) Semisimple rings and modules.
(d) Jacobson's density theorem and the Artin-Wedderburn theorem.
If time allows:
(e) Morita's equivalence (special case); The Brauer group; the Skolem-Noether theorem.

(8) Linear representations of finite groups.
(a) Linear representations of groups.
(b) Maschke's theorem.
(c) Characters. Orthogonality of characters. Frobenius Reciprocity.
(d) Representations of nilpotent groups.
If time allows:
(e) Representations of the symmetric group.
(f) Representations of $\mathrm{GL}_2(\mathbb{F})$, for $\mathbb{F}$ a finite field.
(g) Burnside's p^aq^b theorem.