Course Outline for MATH 570, 2012

Instructor: Prof. Eyal Goren
Method of Evaluation: 30% final project, 70% assignments. Assignments should be typed in LaTeX, and submitted electronically as pdf files.
Text books:

- Dummit and Foote: Abstract algebra. Third edition. John Wiley & Sons, Inc., Hoboken, NJ, 2004.
- Lang: Algebra. Revised third edition. Graduate Texts in Mathematics, 211.
- Jacobson: Basic algebra. I & II. Second edition. W. H. Freeman and Company, New York, 1985.
- Rotman: An introduction to homological algebra. Second edition. Universitext. Springer, 2009.
- Rotman: An introduction to the theory of groups. Fourth edition. Graduate Texts in Mathematics, 148.

You are not required to purchase any of these text-books; They are available at the library. If you wish to purchase one, then I recommend purchasing Dummit and Foote. The next one on my list would be Rotman's book on homological algebra.

Academic integrity: McGill University values academic integrity. Therefore, all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see www.mcgill.ca/integrity for more information).
Submitting work: In accord with McGill Universitys Charter of Students Rights, students in this course have the right to submit in English or in French any written work that is to be graded.
Syllabus and Grade Calculation: In the event of extraordinary circumstances beyond the Universitys control, the content and/or evaluation scheme in this course is subject to change.

The following is an "ideal" syllabus. We will have to see if we can actually cover and what will be deferred to the next term. This depends to a large extent on the students's preparation.

(1) Groups – Selected topics.
  (a) Group actions on sets. (Definition, reformulation as $G \to \Sigma_S$, orbits, stabilizers, Cauchy-Frobenius Formula, revisiting Lagrange's theorem, the coset representation....)
  (b) The class equation, Sylow's theorems, and some basic results on p-groups. Applications (groups of order pq, pq^2, solvability of groups of order less than 60).
  (c) Simplicity of A_n and $\mathrm{PSL}_n(\mathbb{F}_q)$.
  (d) The Jordan-Hölder theorem. Solvable groups.
  (e) Free groups and free products of groups and the concepts of category, functor, adjoint functors. Universal property in terms of initial/final object.
(2) Modules, Part I.
  (a) A recall of the basic definitions and theorems for modules.
  (b) Modules over PID and applications.
  (c) Localization.
  (d) Free modules.
(3) Categories.
  (a) Recall: The language of categories, functors and universal objects.
  (b) Equivalence of categories.
  (c) Injective and projective limits.
(4) Fields.
  (a) Recall: The fundamental theorem of Galois theory and some basic results in Galois theory.
  (b) Profinite groups and, in particular, $\mathbb{Z}_p$.
  (c) Infinite Galois theory.
  (d) Finite fields and cyclotomic fields.
  (e) Kummer theory.
  (f) Solvability by radicals.
  (g) Calculation of Galois groups.
(5) Rings, Part I.
  (a) The spectrum of a ring.
  (b) Integral extensions and the going-up and going-down theorems.
  (c) Noether's normalization lemma, and Hilbert's Nullstellensatz.
  (d) Noetherian and Artinian rings.
  (e) Hilbert's basis theorem.
  (f) Dedekind rings.