MIDTERM EXAM

Elementary Number Theory – Math 392

Instructor: Eyal Goren.

Date: October 21, 1998.
Time: 1 hour, 30 minutes.

Answer the following questions. Notebooks, books and calculators are not allowed. Do NOT use a pencil.

1. Prove that there are infinitely many primes.

2. Prove that $\sqrt[3]{2}$ is irrational.

3. Let d be the greatest common divisor of 33 and 93. Find d. Find x, y such that
$$33x + 93y = d.$$

4. Let
$$\alpha(n) = \sum_{d|n} \mu\left(\frac{n}{d}\right)\mu(d)^2.$$
Find $\alpha(10^a)$ for every $a \in \mathbb{N}$. (Hint: "Think multiplicative").

5. Let ϕ be Euler's function. Prove that $\phi(n) = n - 1$ if and only if n is a prime number.

GOOD LUCK