# ASSIGNMENT 6 – ELEMENTARY NUMBER THEORY MATH 392

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**1**. Prove that a number is rational if and only if its decimal expansion is periodic from some point onward. That is, it is of the form

$$b_n \cdots b_0 \,.\, b_{-1} \cdots b_{-k} a_1 \cdots a_r a_1 \cdots a_r a_1 \cdots a_r \cdots ,$$

for some $a_i, b_i$.

**2**. a. Compute the order of 10 in the group $\mathbb{Z}/p\mathbb{Z}$ for the primes $p = 3, 7, 11, 13, 17$.
b. Write down the decimal expansion of $\frac{1}{3}, \frac{1}{7}, \frac{1}{11}, \frac{1}{13}, \frac{1}{17}$. Phrase a conjecture.
c. Prove your conjecture.

**3**. Prove that there are infinitely many transcendental numbers.

**4**. Prove that for every natural number $b$, the number $x$ defined by

$$[0, 1, b, 1, b, 1, b, 1, b, \cdots]$$

does not satisfy $x^2 \in \mathbb{Q}$.

**5**. a. Write

$$\pi = [a_0, a_1, a_2, \cdots].$$

Using a calculator, compute $a_1, \cdots, a_6$ and the resulting approximations.

**6**. Use quadratic reciprocity to calculate the following

$$\left(\frac{35}{101}\right), \left(\frac{-2}{p}\right), \left(\frac{12}{113}\right).$$

**7**. Find for which primes $p$ we can solve the system of equations

$$x^2 + y^2 = 1, \;\; 3x^2 + 2y^2 = 0.$$