ASSIGNMENT 1 – ELEMENTARY NUMBER THEORY MATH 392

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Answer the following questions:

1. Prove by induction the following formulae:

$$1^2 + 2^2 + 3^2 + \cdots n^2 = \frac{n(n+1)(2n+1)}{6},$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2.$$

2. Prove, using Pascal's triangle, that:
a) $\sum_{m=0}^{n} \binom{n}{m} = 2^n$; b) $\sum_{m=0}^{n} (-1)^n \binom{n}{m} = 0$.

3) Prove, using Pascal's triangle, that: For a fixed k the maximum of $\binom{2k}{m}$ is obtained for $m = k$, and the maximum of $\binom{2k+1}{m}$ is obtained for $m = k$ and $m = k + 1$.

4) a) Prove that if $n \in \mathbb{N}$ is not a square then $\sqrt{n}$ is not rational.
b) Prove that $\sqrt{2} + \sqrt{3}$ is not rational.

5) Do exercises (i) and (vii) from Baker's book (p.7).

6) Pose a new question about primes, and discuss it (finding a complete solution in not required. The question should be new *to you* and the discussion: *your own*).