ASSIGNMENT 10 - MATH 251, WINTER 2007

Submit by Wednesday, April 4, 12:00

1. Let A be a matrix in block form:

$$A = \begin{pmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & & \\ & & \ddots & \\ 0 & 0 & \cdots & A_k \end{pmatrix}.$$

Prove that

$$\Delta_A = \Delta_{A_1}\Delta_{A_2}\cdots\Delta_{A_r},$$

and

$$m_A = \mathrm{lcm}\{m_{A_1}, m_{A_2}, \cdots, m_{A_r}\}.$$

You may use the formula

$$A^b = \begin{pmatrix} A_1^b & 0 & \cdots & 0 \\ 0 & A_2^b & & \\ & & \ddots & \\ 0 & 0 & \cdots & A_k^b \end{pmatrix}$$

for every positive integer b.

2. Let A be an $n \times n$ matrix over an algebraically closed field of characteristic different from 2, such that A^2 is diagonalizable. Prove that if A is a non-singular matrix then also A is a diagonalizable.

3. Prove that the matrix of complex numbers

$$A = \begin{pmatrix} 1 & 1 & 5 & 0 & 0 \\ 7 & 1 & 3 & 0 & 0 \\ 2 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

is diagonalizable. Note: you don't need to diagonalize it to prove it !!

4. Determine the possibilities for the Jordan canonical form of a matrix A with characteristic polynomial $\Delta_A(t) = (t-1)^6(t-2)^4(t-4)^5$ and minimal polynomial $m_A(t) = (t-1)^3(t-2)^2(t-4)$.

5. Find the Jordan canonical form (including the change of basis matrix) of the matrix

$$A = \begin{pmatrix} 5 & 9 & -2 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 & 9 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix}.$$

6. Find a formula for the general term of the sequence

$$0, 1, 4, 12, 32, 80, \ldots$$

$(a_{n+2} = 4a_{n+1} - 4a_n)$.