Written assignment 2, MATH 133 – Vectors, Matrices and Geometry

Submit by Friday, May 28, 12:00. Either submit in class or in the mailbox on the 10-th floor of Burnside Hall.

1. Let A be a diagonalizable matrix and n a positive integer. Prove that A^n is diagonalizable.

2. Let A be an idempotent $n \times n$ matrix. This means that $A^2 = A$. Give 3 examples of such 2×2 matrices. Show that every eigenvalue of A is either 1 or 0.

3. Calculate the following series of determinants. Suggestion: calculate a few examples directly. Then make a guess as to the general formula. Prove the general formula by induction (or any other way).

(1) $\det\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\det\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $\det\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$, $\cdots$

(2) $\det\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $\det\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, $\det\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$, $\det\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$, $\cdots$