Algebra with matrices

Let $A, B, C$ be matrices of size $m \times n$ and $d, e$ scalars. Let 0 be the $m \times n$ zero matrix. We have:

1. $A + B = B + A$;
2. $(A + B) + C = A + (B + C)$;
3. $A + 0 = 0 + A$;
4. $A + (-A) = 0$, where $-A = -1A$;
5. $d(A + B) = dA + dB$;
6. $(d + e)A = dA + eA$;
7. $(de)A = d(eA)$;
8. $1A = A, 0A = 0$ (here 0 on the l.h.s. is a number and on the r.h.s. a matrix).

Let $A, B, C$ be matrices and $d$ a scalar. We assume that the operations below are all well-defined, that is, the sizes of the matrices are such that multiplication or addition makes sense. We have:

1. $(AB)C = A(BC)$;
2. $A(B + C) = AB + AC, (A + B)C = AC + BC$;
3. $d(AB) = (dA)B = A(dB)$;
4. $I_mA = A, A = AI_n$.

We also have:

1. $((A^T)^T = A$;
2. $(dA)^T = dA^T$;
3. $(A + B)^T = A^T + B^T$;
4. $(AB)^T = B^TA^T$. 