Algebra with matrices

Let A, B, C be matrices of size $m \times n$ and d, e scalars. Let 0 be the $m \times n$ zero matrix. We have:

(1) $A + B = B + A$;
(2) $(A + B) + C = A + (B + C)$;
(3) $A + 0 = 0 + A$;
(4) $A + (-A) = 0$, where $-A = -1A$;
(5) $d(A + B) = dA + dB$;
(6) $(d + e)A = dA + eA$;
(7) $(de)A = d(eA)$;
(8) $1A = A, 0A = 0$ (here 0 on the l.h.s. is a number and on the r.h.s. a matrix).

Let A, B, C be matrices and d a scalar. We assume that the operations below are all well-defined, that is, the sizes of the matrices are such that multiplication or addition makes sense. We have:

(1) $(AB)C = A(BC)$;
(2) $A(B + C) = AB + AC, (A + B)C = AC + BC$;
(3) $d(AB) = (dA)B = A(dB)$;
(4) $I_m A = A, A = AI_n$.

We also have:

(1) $((A)^T)^T = A$;
(2) $(dA)^T = dA^T$;
(3) $(A + B)^T = A^T + B^T$;
(4) $(AB)^T = B^T A^T$.