

## Vector bundles on Curves

January 6, 2002

(McGill, Topics in Geometry and Topology III, 189-708 B)

- (1) Goals: Study curves, vector bundles on curves and their moduli.
- (2) Highlights: The Riemann-Roch theorem for vector bundles on curves, Hurwitz's genus formula, Weil's theorem on flat vector bundles, the Hilbert scheme, the Harder-Narasimhan filtration, the construction of the moduli space of semi-stable line bundles on a curve.
- (3) Time and format: 2 consecutive lectures on Tuesday (?), seminar on Thursday afternoon (at Concordia; joint with Prof. Darmon's course).
- (4) Course requirements:
  - Active participation in the seminar. The seminar will be devoted to studying the same concepts studied in the lectures in the context of Riemann surfaces.
  - Final project on an individual topic to be determined together with the student.
- (5) Text books:
  - (a) Hartshorne, Robin: Algebraic geometry. Graduate Texts in Mathematics, No. 52. Springer-Verlag, New York-Heidelberg, 1977.
  - (b) Le Potier: J. Lectures on vector bundles. Cambridge Studies in Advanced Mathematics, 54. Cambridge University Press, Cambridge, 1997.
  - (c) Gunning, R. C.: Lectures on vector bundles over Riemann surfaces. Princeton University Press, Princeton, N.J. 1967
- (6) Syllabus
  - Sheaves on an algebraic variety. The structure sheaf and the sheaf of differentials.
  - Cohomology of Sheaves. Sheaves, resolutions, derived functors, cohomology and hyper-cohomology, Cech cohomology.
  - Vector bundles. Cohomological description. Operations.
  - Curves. The genus of a curve. The Riemann Roch-Theorem. The Hurwitz genus formula.
  - The Hilbert scheme. The Hilbert polynomial and the Grassmannian variety (recollection). The Hilbert scheme.
  - The moduli space of curves.
  - Flat vector bundles. Connections. Flat vector bundles. Construction of flat vector bundles. Weil's Theorem.
  - Semi-stable vector bundles. Stability and semi-stability. Harder-Narasimhan filtration.
  - The moduli space  $M(r, d)$  of (rank  $r$ , degree  $d$ ) semi-stable vector bundles on a curve (modulo GIT).
  - Properties of  $M(r, d)$ . Local deformation theory and the local structure of  $M(r, d)$ .