

BORCHERDS PRODUCTS AND ARITHMETIC APPLICATIONS

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1. TITLES OF THE TALKS (PRELIMINARY)

1. Orthogonal groups and modular forms
 - Orthogonal groups
 - The associated Shimura varieties
 - Special cycles
 - The special cases of signature (1,2), (2,2)
2. Theta liftings of holomorphic and weakly holomorphic modular forms
 - Siegel theta functions
 - The theta lift
 - Weakly holomorphic modular forms and weak Maass forms
 - Regularized theta lifts
3. Borchers products and automorphic Green functions
 - Borchers products
 - Automorphic Green functions
 - Arithmetic Heegner divisors
 - Modularity of Heegner divisors I
4. CM values and Faltings heights
 - A Rankin L -function
 - CM values of automorphic Green functions
 - CM values of Borchers products
 - Faltings heights
5. Harmonic weak Maass forms and the Gross-Zagier formula
 - $X_0(N)$ and $O(1,2)$
 - Modularity of Heegner divisors II
 - The Gross-Zagier formula

2. POSSIBLE COMPLEMENTARY TOPICS (PRELIMINARY)

- (1) The Weil representation and the Siegel Weil formula
- (2) Half-integral weight modular forms and the Shimura lift
- (3) Local and global heights on curves
- (4) Modular curves and Heegner points
- (5) The Gross-Zagier formula and some applications

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Remark by the organizers. The references [Bo1], [BGHZ] (Bruinier’s article), [BY] and [Ku4], may be used as an access point to the literature.

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